	P.R.Government College (Autonomous) KAKINADA	Program&Semester II B.Sc Major & Minor (III Sem) w.e.f 2023-24 admitted batch			
Course Code **MAT – 301 T**	**TITLEOFTHECOURSE** **Group Theory &Problem Solving Sessions**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on sets and number system.	3	1	-	4

Course Objectives:

To provide the learner with the skills, knowledge and competencies to carry out their duties and responsibilities in pure Mathematic environment.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Acquire the basic knowledge and structure of groups
CO2	Get the significance of the notation of a subgroup and cosets.
CO3	Understand the concept of normal subgroups and properties of normal subgroup, permutation and cyclic groups.
CO4	Study the homomorphisms and isomorphisms with applications.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	

UNIT I :

GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II:

SUBGROUPS: Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition-examples-criterion for a complex to be a subgroups; Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset Definition – properties of Cosets – Index of a subgroups of a finite groups – Lagrange's Theorem.

UNIT III:

NORMAL SUBGROUPS:

Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups Sub group of index 2 is a normal sub group.

UNIT IV

HOMOMORPHISM:

Quotient groups, Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

UNIT V:

PERMUTATIONS AND CYCLIC GROUPS:

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups - Definition of cyclic group – elementary properties – classification of cyclic groups.

Co-Curricular Activities

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

TEXT BOOK :

Modern Algebra by A.R.Vasishtha and A.K.Vasishtha, KrishnaPrakashanMedia Pvt. Ltd., Meerut.

REFERENCE BOOKS :

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.

2. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut

3. Rings and Linear Algebra by Pundir&Pundir, published by PragathiPrakashan

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	2	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	2	1	20
II	Subgroups , Co-sets and Lagrange's Theorem	2	1	20
III	Normal subgroups	1	1	15
IV	Homomorphism	1	1	15
V	Permutations and Cyclic Groups	1	2	25
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

…….

Total Marks = 50 M

……..

Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course V: Abstract Algebra
Model Paper(w.e.f. 2024-25)

……

Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions. Selecting at least one question from each part

Part – I 3 X 10 = 30

1. Essay question from Unit -I.
2. Essay question from Unit – II.
3. Essay question from Unit - III.

Part – II

4. Essay question from Unit – IV.
5. Essay question from Unit – V.
6. Essay question from Unit - V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from Unit - I.
8. Short answer question from Unit – I.
9. Short answer question from Unit – II.
10. Short answer question from Unit - II.
11. Short answer question from Unit - III.
12. Short answer question from Unit -IV.
13. Short answer question from Unit - V.

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–V: ABSTRACT ALGEBRA

Short Answer Questions

UNIT-1

1. Prove that in a group the identity element is unique and the inverse of every element is unique.
2. If G is a group,for $a, b \in G$ prove that $(ab)^{-1} = b^{-1}a^{-1}$
3. If every element of a group G is its own inverse, prove that G is abelian.
4. Prove that in a group $G(\neq \emptyset)$, for $a, b, x, y \in G$,the equations $ax = b, ya = b, \forall\, a, b \in G$ have unique solutions.
5. Prove that the group (G, •) is abelian iff $(ab)^2 = a^2 b^2$

UNIT-II

6. If a non -empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
7. If H is a subgroup of a group G then show that HH = H.
8. If H and K are two subgroups of a group G then show that $H \cap K$ is also a subgroup of G.
9. Let h be a subgroup of a group of G and a, b $\in$ G then prove that Ha = Hb iff $ab^{-1} \in H$.

UNIT-III

10. Prove that every subgroup of an abelian group is normal.
11. Show that a subgroup H of a group G is normal iff $xHx^{-1} = H$ for all $x \in G$
12. Prove that intersection of two normal sub-groups of a group is a normal sub-group.
13. Prove that a subgroup of index 2 in a group is a normal subgroup.

UNIT-IV

14. Prove that every quotient group of an abelian group is abelian.
15. If f is a homomorphism of a group G into a group G^1 then show that the kernel of f is a normal subgroup of G.
16. Prove that every homomorphic image of an abelian group is abelian.
17. Let G be a group. If f : G → g defined by $f(x) = x^2$ for all $x \in G$ is a homomorphism then show that G is abelian.

UNIT-V

18. Express the permutation $\begin{pmatrix} 1&2&3&4&5&6&7&8&9 \\ 2&3&4&5&1&6&7&9&8 \end{pmatrix}$ as a product of disjoint cycles.
19. Show that the permutation $\begin{pmatrix} 1&2&3&4&5&6&7&8&9 \\ 6&1&4&3&2&5&7&9&8 \end{pmatrix}$ is odd permutation.
20. Verify whether the permutation $\begin{pmatrix} 1&2&3&4&5&6&7&8&9 \\ 2&5&4&3&6&1&7&9&8 \end{pmatrix}$ is even or odd.
21. Prove that every subgroup of a cyclic group is cyclic.
22. Prove that if G is an infinite cyclic group, then G has exactly two generators.

Essay Questions

UNIT-1

1. Show that the set Q_+ of all positive rational numbers forms an abelian group under the composition defined by 'o' such that $a\ ob = (ab)/3\ \ for\ a, b\ \in\ Q_+$.
2. Prove that the set G of rational numbers other than 1 with operation $*$ such that a $*$ b = a + b – ab for all a, b $\in$G is an abelian group.
3. Prove that the set of n^{th} roots of unity forms an abelian group w.r.t. '.'
4. Prove that a finite semi-group (G , •) satisfying the cancelation laws is a group.

UNIT-II

5. Prove that a non-empty complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G.
6. If H and K are two sub-groups of a group G , then H $\cup$ K is a subgroup iff either H $\subseteq$ K or K $\subseteq$ H.
7. If H and K are two sub-groups of a group G , then show that HK is a sub-group of G if and only if HK = KH.
8. State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true.
9. Prove that any two left cosets of a subgroup are either disjoint or identical.

UNIT-III

10. Prove that H of a group G is normal sub-group of G if and only if each left coset of H in G is a right coset of H in G.
11. Prove that H is a normal sub-group of G if and only if product of two right right (left) cosets of H in G is again a right (left) coset of H on G .
12. State and prove Fundamental theorem of homomorphism of groups.
13. If M, N are two normal subgroups of a group G such that M$\cap$N = {e}, then show that every element of M commutes with every element of N.

UNIT-IV

14. Prove that the set G/H of all cosets of a normal subgroup H in a group G with respect to coset multiplication is a group.
15. State and prove fundamental theorem of homomorphism of groups.
16. Show that the necessary and sufficient condition for a homomorphism f of a group G onto a group G^1 with kernel K to be an isomorphism of G into G^1 is that K = {e}.

UNIT-V

17. Let S_n be a symmetric group of n symbols and let A_n be the group of even permutations, then show that A_n is a normal in S_n and $O(A_n) = \frac{n!}{2}$.
18. State and prove Cayley's theorem.
19. Prove that every subgroup of a cyclic group is cyclic.
20. Prove that the order of a cyclic group is equal to the order of its generators.
21. Prove that a cyclic group of order n has $\varphi(n)$ generators.

	P.R.Government College (Autonomous) KAKINADA	Program&Semester II B.Sc Major (III Sem) w.e.f 2023-24 admitted batch			
CourseCode MAT-302 T	**TITLEOFTHECOURSE** **Numerical Methods &Problem Solving Sessions**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Advanced Calculus, Linear Algebra and Differential Equations	3	1	-	4

Course Objectives:

This course will cover the classical fundamental topics in numerical methods such as, approximation, finite differences, Interpolation with equal and unequal intervals, solution of Algebraic and Transcendental equations and Curve fitting.

Course Outcomes:

On Completion of the course, the students will be able o-	
CO1	Difference between the operators, Δ, ∇, E and the relation between them.
CO2	Know about the Newton – Gregory Forward and backward interpolation, Central Difference operators, δ, μ, σ and relation between them
CO3	Solve Algebraic and Transcendental equations.
CO4	Understand the concept of Curve fitting.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	

Unit – 1:

The calculus of finite differences

The operators,Δ, ∇, E - Fundamental theorem of difference calculus- properties of, Δ, ∇, E and problems on them to express any value of the function in terms of the leading terms and the leading differences - relations between E and D - relation between D and Δ - problems on one or more missing terms- Factorial notation- problems on separation of symbols- problems on Factorial notation.

Unit – 2: Interpolation with Equal and Unequal intervals

Derivations of Newton – Gregory Forward and backward interpolation and problems on them.

Divided differences - Newton divided difference formula - Lagrange's and problems on them.

Unit – 3: Central Difference Interpolation formulae

Central Difference operators, δ, μ, σ and relation between them - Gauss forward formula for equal intervals - Gauss Backward formula - Stirlings formula - Bessel's formula and problems on the above formulae.

Unit – 4: Solution of Algebraic and Transcendental equation

Method for finding initial approximate value of the root - Bisection method - to find the solution of given equations by using (i) Regula Falsi method (ii) Iteration method (iii) Newton – Raphson's method and problems on them.

Unit – 5: Curve Fitting

Least-squares curve fitting procedures - fitting a straight line-nonlinear curve fitting-curve fitting by a sum of exponentials.

Text Book

Numerical Analysis by G. Shanker Rao, New Age International Publications

References:

1. Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, Pearson,(2003) 7^{th} Edition.

2.Introductory Methods of Numerical Analysis by S.S. Sastry, (6th Edition) PHI New Delhi 2012

3. Numerical Methods for Scientific and Engineering Computation by M. K. Jain, S .R. K. Iyengar and R. K. Jain, New Age International Publishers (2012), 6^{th} edition.

Co-Curricular Activities:

Seminar/ Quiz/ Assignments/ Applications of Numerical methods to Real life Problem /Problem Solving Sessions.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	3	2	3	3	2	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-III : PAPER-VI

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	The calculus of finite differences	2	1	20
II	Interpolation with Equal and Unequal intervals	2	2	30
III	Central Difference Interpolation formulae	1	1	15
IV	Solution of Algebraic and Transcendental equation	1	1	15
V	Curve fitting	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 3X10 = 30

…….

Total Marks = 50

……

Pithapur Rajah's Government College (Autonomous), Kakinada

II year B.Sc., Degree Examinations - III Semester

Mathematics Course VI: NUMERICAL METHODS

Model Paper (w.e.f. 2024-25)

……

Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer Any Three Questions, Selecting At Least One Question from Each Part.

Part – A **3 X 10 = 30 M**

1. Essay question from unit - I.
2. Essay question from unit – II.
3. Essay question from unit – II.

Part – B

4. Essay question from unit - III.
5. Essay question from unit - IV.
6. Essay question from unit - V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit – I.
9. Short answer question from unit – II.
10. Short answer question from unit – II.
11. Short answer question from unit - III.
12. Short answer question from unit - IV.
13. Short answer question from unit – V.

***

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank for
PAPER–: NUMERICAL METHODS

Short Answer Questions

Unit-I

1. Prove that i) $\Delta = E - 1$ ii) $\nabla = 1 - E^{-1}$
2. Prove that i) $(1 + \Delta)(1 - \nabla) = 1$ ii) $E\nabla = \Delta$ iii) $\Delta - \nabla = \Delta\nabla$
3. Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$, (ii) $\Delta = \frac{\delta^2}{2} + \delta \sqrt{1 + \frac{\delta^2}{4}}$
4. Prove that i) $u_3 = u_2 + \Delta u_1 + \Delta^2 u_0 + \Delta^3 u_0$ ii) $u_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$
5. Given $y_0 = 3$, $y_1 = 12$, $y_3 = 81$, $y_4 = 100$. Find $\Delta^4 y_0$ without forming difference table .
6. Find the missing term in the following data.

x	0	1	2	3	4
y	1	3	9	?	81

UNIT - II

7. Compute f(1.1) from the following data.

x	1	2	3	4	5
f(x)	7	12	29	64	123

8. From the following table find y value at x = 0.26

x	0.10	0.15	0.20	0.25	0.30
y = Tanx	0.1003	0.1511	0.2027	0.2553	0.3093

9. Show that $f (x_0 , x_1 , x_2 , \dots , x_n) = \frac{\Delta^n f(x_0)}{n! h^n}$.
10. Find the third divided difference with arguments 2 , 4 , 9 , 10 of the function $f(x) = x^3 - 2x$.
11. By Lagrange's interpolation formula , find the value of y at x = 5 , given that

x	1	3	4	8	10
f(x)	8	15	19	32	40

12. Using Lagrange's interpolation formula , prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})]$$

UNIT – III

13. Using Gauss forward formula find u_{30} from the given data u_{21} = 18.4708, u_{25} = 17.8144, u_{29} = 17.1070, u_{33} = 16.3432, u_{37} = 15.5154.
14. Given that $\sqrt{12500} = 111.803399$, $\sqrt{12510} = 111.848111$, $\sqrt{12520} = 111.892806$, $\sqrt{12530} = 111.937483$, show $\sqrt{12516} = 111.8749301$ by using Gauss backward interpolation formula.
15. State and prove Stitling's formula
16. Apply Stirling's formula to find y_{28} given that y_{20}=49225, $y_{25} = 48316$, $y_{30} = 47236$, $y_{35} = 45926$, $y_{40} = 44300$.
17. Given y_{20} = 24 , y_{24} = 32 , y_{28} = 35 , y_{32} = 40 , find y_{25} by Bessel's formula .

Unit - IV

18. Find a real root of the equation $x^3 - 6x - 4 = 0$ by bisection method.
19. Find a real root of the equation $x^3 - x - 1 = 0$ by bisection method.
20. Find the root of the equation $x^3 + x^2 - 1 = 0$ by iteration method.
21. Find the square root of 2.
22. Find a real root of the equation $x = e^{-x}$, using the Newton – Raphson method.

UNIT - V

23. Obtain the normal equations to the least square line y = a + bx .
24. Find the least square line y = a + bx and y(5) for the data.

x	0	2	5	7
y	-1	5	12	20

25. Find the least square line For the data points (-1, 10), (0, 9), (1, 7), (2, 5), (3, 4), (4, 3), (5, 0) and (6, -1).
26. Fit a polynomial of the second degree to the data points

x	0	1	2
y	1	6	17

27. Fit the exponential curve $y = ae^{bx}$ to the following data.

x	2	4	6	8
y	25	38	56	84

Unit - I

1. State and prove fundamental theorem of Difference calculus.
2. Show that $\Delta^n \cos(ax+b) = (2sin\frac{ah}{2})^n \cos[a+bx+n\left(\frac{ah+\pi}{2}\right)]$
3. Obtain the estimate of the missing terms in the following data.

x	1	2	3	4	5	6	7	8
f(x)	1	8	?	64	?	216	343	512

4. Prove that $\Delta^n x^{(n)} = n!\, h^n \; and \; \Delta^{n+1} x^{(n)} = 0$

UNIT - II

5. State and prove Newton's – Gregory formula for forward interpolation with equal intervals .
6. The area of a circle of diameter d is given for the following values , find the approximate value for the area of a circle of diameter 82 .

d(Diameter)	80	85	90	95	100
A(Area)	5026	5674	6362	7088	7854

7. From the following table , find the number of students who obtain less than 56 marks .

Marks	30-40	40-50	50-60	60-70	70-80
No.of students	31	42	51	35	31

8. Given

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

Find f(7.5)

9. The population of a country in the decennial census were as under . Estimate the population for the year 1925 .

Year(x)	1891	1901	1911	1921	1931
Population(y) (in thousands)	46	66	81	93	101

10. State and prove Netown's divided difference formula .

11. By means of Newton's divided difference formula, find the values of $f(8), f(15)$ from the following table.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

UNIT - III

12. Using Gauss forward formula find u_{32} from the given data $u_{20} = 14.035$, $u_{25} = 13.674$, $u_{30} = 13.257$, $u_{35} = 12.734$, $u_{40} = 12.089$, $u_{45} = 11.309$.
13. Apply Gauss forward formula to find the value of u_9 if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{16} = 40$.
14. Interpolate by means of Gauss backward interpolation formula the sales for the concern for the year 1936, given that

$year$	1901	1911	1921	1931	1941	1951
$sales(in\ thousands)$	12	15	20	27	39	52

15. Apply Stirling's formula to find a polynomial of degree four which takes

x	1	2	3	4	5
y	1	-1	1	-1	1

16. Apply Bessel's formula to obtain find y_{25} given that y_{20}=2854, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

UNIT - V

17. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position up to three places of decimals.
18. Find a real root of the equation $x^3 - x - 4 = 0$ correct to three decimal places by the method of Regula - False position.
19. Find a real root of the equation $\cos x = 3x - 1$, correct to three decimal places, using iteration method.
20. Solve $x = 0.21 \sin(0.5 + x)$ by iteration method starting with $x = 0.12$.
21. Find the real root of the equation $x^2 - 5x + 2 = 0$ by Newton- Raphson's method.

22. Using Newton -Raphson method, establish the iterative formula $x_{n+1} = \frac{1}{3}(2x_n + \frac{N}{x_n^2})$ to calculate the cube root of N and hence find the cube root of 12.

UNIT - V

23. Obtain the normal equations to the parabola $y = a + bx + cx^2$ by using least square method.
24. Fit a second-degree parabola to the following data.

x	0	1	2	3	4
y	1	5	10	22	38

25. Determine the constants a and b by the least squares method such that $y = ae^{bx}$, fits the following data.

x	1.0	1.2	1.4	1.6
y	40.170	73.196	133.372	243.02

26. Fit a curve of the form $y = ax^b$ to the following data.

x	2	4	6	8	10
y	0.973	3.839	8.641	15.987	23.794

27. Fit a curve of the form $y = ab^x$ to the following data.

x	1	2	3	4
y	4	11	35	100

	P.R.Government College (Autonomous) **KAKINADA**	**Program&Semester** II B.Sc. Major (III Sem) w.e.f 2023-24 admitted batch			
CourseCode MAT- 303 T	**TITLEOFTHECOURSE** **Laplace Transforms & Problem Solving Sessions**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Knowledge of Calculus, specifically integration and differentiation and an understanding of Complex numbers.	3	1	-	3

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the definition and properties of Laplace transformations.
CO2	Get an idea about first and second shifting theorems and change of scale property.
CO3	Understand Laplace transforms of standard functions like Bessel, Error function etc
CO4	Know the reverse transformation of Laplace and properties.
CO5	Get the knowledge of application of convolution theorem.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

UNIT I

LAPLACE TRANSFORMS – I

Definition of Laplace Transform - Linearity Property - Piecewise Continuous Function - Existence of Laplace Transform - Functions of Exponential order and of Class A.

UNIT II:

LAPLACE TRANSFORMS – II

First Shifting Theorem, Second Shifting Theorem, Change of Scale Property, Laplace transform of the

derivative of f(t), Initial value theorem and Final value theorem.

UNIT III:
LAPLACE TRNASFORM – III

Laplace Transform of Integrals - Multiplication by t, Multiplication by tn - division by t -Laplace transform of Bessel Function - Laplace Transform of Error Function -Laplace transform of Sine and Cosine integrals

UNIT IV:

INVERSE LAPLACE TRANSFORMS – I

Definition of Inverse Laplace Transform - Linearity Property - First Shifting Theorem -Second Shifting Theorem - Change of Scale property - use of partial fractions - Examples.

UNIT V:

INVERSE LAPLACE TRANSFORMS – II

Inverse Laplace transforms of Derivatives - Inverse Laplace Transforms of Integrals -Multiplication by Powers of 'p' - Division by powers of 'p' - Convolution Definition -Convolution Theorem - proof and Applications - Heaviside's Expansion theorem and its Applications.

Co-Curricular Activities

Seminar/ Quiz/ Assignments/ Applications of Laplace Transforms to Real life Problem /Problem Solving Sessions.

TEXT BOOK:

LaplaceTransforms by A.R.Vasishtha,Dr.R.K.Gupta,KrishnaPrakashanMedia Pvt.Ltd., Meerut.

REFERENCE BOOKS:

1. Introduction to Applied Mathematics by Gilbert Strang, Cambridge Press

2. Laplace and Fouries transforms by Dr.J.K. Goyal and K.P. Guptha, PragathiPrakashan, Meerut.

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	1	3	3	1	1	3	1	2
CO5	2	2	3	2	2	3	3	1	3	3	3	2	1

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	LAPLACE TRANSFORMS – I	1	1	15
II	LAPLACE TRANSFORMS – II	2	2	30
III	LAPLACE TRNASFORM – III	2	1	20
IV	INVERSE LAPLACE TRANSFORMS – I	1	1	15
V	INVERSE LAPLACE TRANSFORMS – II	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

…….

Total Marks = 50 M

……..

Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course VII: LAPLACE TRANSFORMS
Model Paper (w.e.f. 2024-25)

……

Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions selecting atleast one question from each part

Part – A **3 X 10 = 30**

1. Essay question from unit - I.
2. Essay question from unit - II.
3. Essay question from unit - II.

Part – B

4. Essay question from unit - III.
5. Essay question from unit - IV.
6. Essay question from unit - V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit - II.
9. Short answer question from unit – II.
10. Short answer question from unit - III .
11. Short answer question from unit – III.
12. Short answer question from unit – IV.
13. Short answer question from unit - V

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–VII : LAPLACE TRANSFORMS

Short answers

Unit - I

1. Find the Laplace transform of $(t^2+1)^2$
2. Find the Laplace transform of $e^{2t}+4t^3-2\,sin3t+3\,cos3t$
3. Find $L\{7e^{2t}+9e^{-2t}+5\,cost+7t^3+5\,sin3t+2\}$
4. Find the L{F(t)} where $F(t)=\begin{cases}e^{t-a}, t>a\\ 0\,,\ t<a\end{cases}$
5. Prove that the function F(t) = t^2 is of exponential order 3.

Unit - II

6. Find the Laplace transform of $e^{-t}(3\,sin2t-5\cosh 2t)$.
7. Find $L\{(t+3)^2e^t\}$
8. Find $L\{(1+te^{-t})^3\}$
9. Find the Laplace transform of G(t), where $G(t)=\begin{cases}\cos(t-\frac{\pi}{3}), t>\frac{\pi}{3}\\ 0\,,\ t<\frac{\pi}{3}\end{cases}$
10. Find the Laplace transform of $e^{-3t}u(t-2)$
11. State and prove Change of Scale property.
12. Apply change of scale property, if $L\{F(t)\}=\frac{p^2-p-1}{(2p+1)^2(p-1)}$.
13. If $L\{sin\sqrt{t}\}=\frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}e^{\frac{-1}{4p}}$, find $L\left\{\frac{cos\sqrt{t}}{\sqrt{t}}\right\}$.

Unit – III

14. Find $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$
15. Find $L\int_0^t\int_0^t \cosh au\ du\,du$
16. Evaluate $L\{\sin at-at\cos at\}$
17. Find $l\{\ te^{3t}\sin 2t\ \}$
18. Find the Laplace transform of $\frac{e^{-at}-e^{-bt}}{t}$

19. Find the Laplace transform of $\frac{\sin at}{t}$

20. Show that $L\left\{\frac{\cosh at}{t}\right\}$ does not exists.

21. Prove that $L\{J_1(t)\} = 1 - \frac{p}{\sqrt{p^2+1}}$

Unit – IV

22. State and prove second shifting theorem

23. State and prove change of scale property.

24. Find $L^{-1}\left[\frac{3p-2}{p^{\frac{5}{2}}} - \frac{7}{3p+2}\right]$

25. Find $L^{-1}\left[\frac{1}{(p+1)(p-2)}\right]$

26. Prove that $L^{-1}\left[\frac{p^2}{(p+2)^3}\right] = e^{-2t}(1 - 4t + 2t^2)$

27. Find $L^{-1}\left[\frac{e^{-5p}}{(p-2)^4}\right]$

28. If $L^{-1}\left[\frac{p}{(p^2+1)^2}\right] = \frac{1}{2}t \sin t$, find $L^{-1}\left[\frac{8p}{(4p^2+1)^2}\right]$

Unit – V

29. Evaluate $L^{-1}\left[\frac{p}{(p^2+a^2)^2}\right]$

30. Find $L^{-1}\left[\frac{p}{(p^2-a^2)^2}\right]$

31. Find $L^{-1}\left[\frac{p}{p^2-a^2}\right]$

32. Find $L^{-1}\left[\frac{1}{p}\log(\frac{p+2}{p+1})\right]$

33. Find the inverse Laplace transform of $\frac{1}{p^3(p^2+1)}$

Essay Question

Unit - I

1. Find the Laplace transform of F(t) = | t – 1 | + | t + 1 |, t ≥ 0.
2. Find the Laplace Transform of $(\sin t - \cos t)^3$.
3. Obtain the Laplace transform the function $F(t) = \begin{cases}(t-1)^2, t > 1 \\ 0\,,\ 0 < t < 1\end{cases}$
4. Find the Laplace Transform of $F(t) = \begin{cases} 0,\ t > \pi \\ \sin t\,,\ 0 < t < \pi\end{cases}$

5. Using the expansion $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$, show that $L\{sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2p^{\frac{3}{2}}} e^{\frac{-1}{4p}}$

Unit - II

6. State and prove First Shifting theorem.

7. State and prove Second Shifting theorem.

8. Find $L\{\sinh at \cos at\}$

9. Prove that L{ $F^{(n)}(t)$ } = p^n f(p) – $p^{n-1}F(0)$- $p^{n-2}F^1(0)$-.....-$F^{(n-1)}(0)$.

10. State and prove Initial Value theorem.

11. State and prove Final value theorem.

12. Show that $(i)L\{t\ sinat\} = \frac{2ap}{(p^2+a^2)^2}$ $and (ii)L\{t\ cosat\} = \frac{p^2-a^2}{(p^2+a^2)^2}$

13. If $L\{t\ sinat\} = \frac{2ap}{(p^2+a^2)^2}$, then prove that $L\{\sin at + at\ cosat\} = \frac{2ap}{(p^2+a^2)^2}$

Unit – III

14. Find $L\{(t^2 - 3t + 2)\sin 3t\}$

15. Find $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$.

16. Using Laplace transform, evaluate $\int_0^\infty \frac{\cos at - \cos abt}{t} dt$

17. Find $L\left(\text{erf}\left(\sqrt{t}\right)\right)$ and hence prove that $L\left(t\ \text{erf}\left(2\sqrt{t}\right)\right) = \frac{3p+8}{p^2(p+4)^{\frac{3}{2}}}$

18. Prove that $L\{J_0(t)\} = \frac{1}{\sqrt{p^2+1}}$

19. Find the Laplace transform of $S_i(t)$.

20. Find the Laplace transform of $C_i(t)$.

Unit – IV

21. Find $L^{-1}\left[\frac{3}{p^2-3} - \frac{3p+2}{p^3} - \frac{3p-27}{p^2+9} + \frac{6-30\sqrt{p}}{p^4}\right]$

22. Find $L^{-1}\left[\frac{3p+1}{(p-1)(p^2+1)}\right]$

23. Find the inverse Laplace transform of $\left[\frac{4p+5}{(p-1)^2(p+2)}\right]$

24. Find $L^{-1}\left[\frac{e^{-\pi p}(p+1)}{p^2+p+1}\right]$

25. For a > 0, prove that $L^{-1}\{f(p)\} = F(t) implies\ that\ L^{-1}[f(ap+b)] = \frac{1}{a} e^{\frac{-bt}{a}} F\left(\frac{t}{a}\right)$

Unit - V

26. Find $L^{-1}\left[\frac{p-3}{p^2+4p+13}\right]$

27. Using Convolution theorem, find $L^{-1}\left[\frac{p}{(p^2+a^2)^2}\right]$

28. Apply Convolution theorem to find $L^{-1}\left[\frac{p+1}{(p^2+2p+2)^2}\right]$

29. Apply Heaviside's expansion formula to find $L^{-1}\left[\frac{6p^2+22p+18}{p^3+6p^2+11p+6}\right]$

30. Using Heaviside's expansion formula, find $L^{-1}\left[\frac{3p+1}{(p-1)(p^2+1)}\right]$

***

	P.R.Government College (Autonomous) **KAKINADA**	**Program&Semester** II B.Sc. Major (III Sem) w.e.f 2023-24 admitted batch			
Course Code MAT- 304 T	**TITLEOFTHECOURSE** **Special Functions & Problem Solving Sessions**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Multivariable calculus and Differential Equations	3	1	-	3

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations.
CO2	Find power series solutions of ordinary differential equations
CO3	Solve Hermite equation and write the Hermite Polynomial of order (degree) n, also
CO4	Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials.
CO5	Solve Bessel equation and write the Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel unction.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

UNIT I

Beta and Gamma functions, Chebyshev polynomials

Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.

Another form of Beta Function, Relation between Beta and Gamma Functions.

Chebyshev polynomials, orthogonal properties of Chebyshev polynomials, recurrence relations, generating functions for Chebyshev polynomials.

UNIT II:

Power series and Power series solutions of ordinary differential equations.

Introduction, summary of useful results, power series, radius of convergence, theorems on Power series Introduction of power series solutions of ordinary differential equation Ordinary and singular points, regular and irregular singular points, power series solution.

UNIT III:

Hermite polynomials

Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

UNIT IV:

Legendre polynomials

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials. ,

2. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required)to show that $P_n(x)$ is the coefficient of h^n , in the expansion of $(1 - 2xh + h^2)^{-1/2}$

3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

UNIT V:

Bessel's equation

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.

2. Integration of Bessel's equation in series form=0, Definitionof $J_n(x)$, recurrence formulae for $J_n(x)$. 3. Generating function for $J_n(x)$, orthogonally of Bessel functions.

Co-Curricular Activities

Seminar/ Quiz/ Assignments/ Applications of Functions of complex variables to Real life Problem /Problem Solving Sessions.

TEXT BOOK:

Theory of Functions of a Complex variable by Shanti Narayan &Dr. P. K. Mittal, S. Chand &Company Ltd.

REFERENCE BOOKS:

1. Theory of Functions of a Complex Variable by A. I. Markushevich, Second Edition, AMS Chelsea Publishing

2. Theory And Applications by M. S. Kasara, Complex Variables, 2nd Edition, Prentice Hall India Learning Private Limited

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	1	3	3	1	1	3	1	2
CO5	2	2	3	2	2	3	3	1	3	3	3	2	1

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Beta and Gamma functions, Chebyshev polynomials.	2	2	15
II	Power series and Power series solutions of ordinary differential equations.	2	1	30
III	Hermite polynomials	1	1	20
IV	Legendre polynomials	1	1	15
V	Bessel's equation	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

…….

Total Marks = 50 M

……..

Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course VIII: SPECIAL FUNCTIONS
Model Paper (w.e.f. 2024-25)

……

Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions selecting atleast one question from each part

Part – A **3 X 10 = 30**

1. Essay question from unit - I.
2. Essay question from unit - I.
3. Essay question from unit - II.

Part – B

4. Essay question from unit - III.
5. Essay question from unit - IV.
6. Essay question from unit - V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit - I.
9. Short answer question from unit – II.
10. Short answer question from unit - II .
11. Short answer question from unit – III.
12. Short answer question from unit – IV.
13. Short answer question from unit - V

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA

II B.SC MATHEMATICS MAJOR – Semester V (w.e.f. 2024-2025)

Mathematics Course VIII: SPECIAL FUNCTIONS

QUESTION BANK

Short Answer questions

Unit - I

1. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$
2. Evaluate $\int_0^1 \frac{dx}{\sqrt{-log_e x}}$
3. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}} dy$ *and hence show that* $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
4. Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$
5. Prove that $(1 - x^2)T^1_n(x) = - n\ x\ T_n(x) + n\ T_{n-1}(x)$
6. Prove that $U_{n+1}(x) - 2x\ U_n(x) + U_{n-1}(x) = 0$
7. Find the first four Chebyshev polynomials.
8. Prove that $T_n(-1) = (-1)^n$ and $U_n(-1) = 0$

Unit – II

9. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$.
10. Find the radius of convergence of the series $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$
11. Find the radius of the convergence of the series $\sum(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
12. Determine whether x = 0 is an ordinary point or a regular singular point of the differential equation $2x^2\left(\frac{d^2y}{dx^2}\right) + 7x(x+1)\frac{dy}{dx} - 3y = 0.$
13. Show that x = 0 and x = -1 are singular points of $x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0$ where the first is irregular and the other is regular.
14. Solve by power series method $y' - y = 0$.

Unit - III

15. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ *and* $H_{2n+1}(0) = 0.$
16. Find Hermit Polynomials for n=0, 1, 2, 3, 4 .
17. Prove that $H_n'' = 4n(n-1)H_{n-2}$
18. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$
19. Prove that $H_n(-x) = (-1)^n H_n(x)$.
20. Prove that, if $m < n$, $\frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^m n!}{(n-m)!}H_{n-m}(x).$

Unit - IV

21. Prove that $P_n(-x) = (-1)^n P_n(x)$ *and hence deduce that* $P_n(-1) = (-1)^n$
22. Prove that $P_n' = \frac{n(n+1)}{2}$
23. Prove that $(2n+1)P_n = P'_{n+1} - P'_{n-1}$.
24. Prove that $xP_n' - P'_{n-1} = nP_n$.
25. Prove that $(n+1)P_n = P'_{n+1} - xP_n'$.
26. Prove that $(1-x^2)P_n' = n(P_{n-1} - xP_n)$.
27. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

Unit – V

28. Prove that, when n is a positive integer $J_{-n}(x) = (-1)^n J_n(x)$.

39. Show that $J_n(-x) = (-1)^n J_n(x)$ for positive or negative integers.

30. Prove that $J_n(x) = \frac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$

31. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

32. Prove that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\, sin\theta)\, d\theta$

33. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

34. Show that $\int_0^{\infty} e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{(a^2+b^2)}}$, $a > 0$

Essay Questions

Unit –I

1. When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.5.....(2n-1)}$

2. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

3. Prove that $\int_0^{\pi/2} sin^{2l-1}\theta . cos^{2m-1}\theta\, d\theta = \frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

4. Evaluate $i) \int_0^a \frac{dx}{(a^n - x^n)^{\frac{1}{n}}}$ $ii) \int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx$

5.State and prove orthogonal properties of Chebyshev polynomials.

6. prove that $T_n(x)$ and $U_n(x)$ are independent solution of Chebyshev's differential equation.

7. Show that $\frac{1}{\sqrt{1-x^2}}U_n(x)$ satisfies the differential equation $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (n^2-1)y = 0$.

Unit – II

1. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$.
2. Find the radius of convergence the exact interval of convergence of the power series $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$
3. Determine the interval of convergence of the power series $\sum\{\frac{1}{n}(-1)^{n+1}(x-1)^n\}$
4. Find the power series solution of the equation $(x^2+1)y'' + xy' - xy = 0$ in powers of x.
5. Find the solution in series of $\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right) + x^2y = 0$ about x = 0 .
6. Find the general solution of $y'' + (x-3)y' + y = 0$ near x = 2 .

Unit – III

1. State and Prove generating function of the Hermit's polynomial.
2. State and Prove Rodrigues formula for $H_n(x)$.
3. State and Prove Orthogonal Properties of Hermite Polynomials.
4. Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.
5. Prove that $H_n'(x) = 2nH_{n-1}(x) \quad n \geq 1$ and $H_0'(x) = 0$.
6. Prove that $H_n(x) = (-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$

Unit – IV

1. Show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1-2xh+h^2)^{-1/2}$ in Ascending powers of h for $|x| \leq 1$ and $|h| < 1$.
2. Prove that $P_n(x) = \frac{1}{n!2^n}.\frac{d^n}{dx^n}(x^2-1)^n$.
3. Prove that $\int_{-1}^{1}[P_n(x)]^2 dx = \frac{2}{2n+1}$.
4. Prove that $\int_{-1}^{1} P_m(x).P_n(x)dx = 0 \ \ if\ m \neq n.$ and 2/(2n+1) if m = n .
5. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$
6. Prove that $\int_{-1}^{1}(x^2-1)P_{n+1}P_n'\,dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$.

Unit V

1. Prove that when n is a positive integer, $J_n(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x(z-\frac{1}{z})}{2}}$ in ascending and descending powers of z.
2. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

3. Prove that $xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$.

4. Prove that $x^2 J''_n(x) = (n^2 - n - x^2)J_n(x) + xJ_{n+1}(x)$

5. Prove that $i)\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ $(ii)\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$.

6. Prove that $\sqrt{\frac{\pi x}{2}} J_{3/2}(x) = \frac{1}{x}\sin x - \cos x$.

****